Code No: 152AA **R18**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year II Semester Examinations, January/February - 2024
MATHEMATICS - II
(Common to CSE, IT, CSIT, ITE, CE(SE), CSE(CS), CSE(DS), CSE(N), CSD)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Justify whether the following differential is exact or not?
$(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0.$ [2]

b) Write the solution of Bernoulli's differential equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$. [3]

c) The general form of a differential equation is given by:
$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} + \cdots + a_n y = V$ or $[f(D)]y = V$.
Where $a_1, a_2, \cdots, a_n$ are constants and V is a function of x only. Then, write down the expression for the general solution of this differential equation $[f(D)]y = 0$.
When all the roots of f (m) = 0 are real and different. [2]

d) Reduce the following Cauchy-Euler equation to linear ODE with constant coefficients.
$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 3y = 0.$ [3]

e) Evaluate the double integral, $\iint_R 6xy^2 dA$ over the rectangular region bounded by the lines x = 2, y = 1, x = 4, y = 2. [2]

f) Find the volume of the region D bounded by planes
$x = 0, y = 0, z = 0, x = a, y = b, z = c$ using triple integration. [3]

g) Find the unit normal to the surface $xy^3z^2 = 4$ at $(-1, -1, 2)$. [2]

h) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ, where Q is the point $(5, 0, 4)$. [3]

i) If a force $\vec{F} = 2x^2y\bar{i} + 3xy\bar{j}$ displaces a particle in the xy-plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$. Find the work done. [2]

j) Evaluate $\iint_S (yz\bar{i} + zx\bar{j} + xy\bar{k}) \cdot \overrightarrow{ds}$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. [3]

PART - B

(50 Marks)

2.a) Solve $(2x\,log\,x - xy)dy + 2ydx = 0$.

b) Solve $x^3p^2 + x^2yp + a^3 = 0$, where $p = \frac{dy}{dx}$. [5+5]

OR

3.a) The rate at which a body cools is proportional to the difference between the temperature of the body and that of the surrounding air. If a body in air at 25^oC will cool from 100°C to 75°C in one minute, find its temperature at the end of three minutes.

b) Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$. [5+5]

4.a) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$.

b) Apply method of variation of parameters to solve: $\frac{d^2y}{dx^2} + y = Cosec\ x$. [5+5]

OR

5.a) Apply method of variation of parameters to solve: $\frac{d^2y}{dx^2} + y = \tan x$.

b) Find the general solution of $(D^2 + D)y = x^2 + 2x + 4$. [5+5]

6.a) Evaluate $\iint (x+y)^2 dxdy$ over the region bounded by the ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

b) Find the mass of the region in the first octant bounded by $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. [5+5]

OR

7.a) Evaluate $\iint (x^2 + y^2) dxdy$ throughout the area enclosed by curves $y = 4x, x + y = 3, y = 0$, and $y = 2$.

b) Find the co-ordinates of the centre of gravity of the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$, density being given by $\rho = xyz$. [5+5]

8. Show that the vector field $\vec{F} = \frac{\vec{r}}{|\vec{r}|^3}$ is irrotational as well as solenoidal. Find the scalar potential. [10]

OR

9.a) Given the vector field $\vec{V} = (x^2 - y^2 + 2xz)\bar{i} + (xz - xy + yz)\bar{j} + (z^2 + x^2)\bar{k}$, find curl$(\vec{V})$. Do the vectors given by curl$(\vec{V})$ at $P_0(1, 2, -3)$ and $P_1(2, 3, 12)$ orthogonal?

b) Find the constants a, b, c so that, $\vec{F} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$, is irrotational and hence find function ϕ such that $\vec{F} = \nabla\phi$. [5+5]

10. Verify the divergence theorem for the function $\vec{F} = 2x^2y\bar{i} - y^2\bar{j} + 4xz^2\bar{k}$, taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$. [10]

OR

11. Verify Stoke's theorem for the function $\vec{F} = z\bar{i} + x\bar{j} + y\bar{k}$, where C is the unit circle in the xy-plane bounding the hemisphere $z = \sqrt{1 - x^2 - y^2}$. [10]

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